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NEW APPROACH TO THE DESIGN OF DIGITAL ALGORITHMS FOR ELECTRIC POWER MEASUREMENTS

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Abstract – This paper introduces a new approach to the design of digital algorithms for electric power measurements. Digital algorithms for electric power measurements are represented as 2D digital FIR filters applied on voltage and current samples. Based on this approach, a new technique for algorithm design is developed. As the main advantage, the technique provides a convenient way to design new algorithms for measuring the electric power components according to various definitions in both sinusoidal and non-sinusoidal conditions. Several new algorithms are derived by using the proposed design technique. The existing algorithms for power measurements are also derived by using the new approach. The algorithm performance is tested using actual signal recordings.

Keywords: Electric power measurements, Active and reactive power, Digital signal processing.

INTRODUCTION

Power components in electric power systems with sinusoidal, non-sinusoidal linear, and non-sinusoidal nonlinear circuits are to be measured for various purposes such as revenue metering, power factor compensation, and control. While definitions of power components are well known in sinusoidal systems, there are no universally accepted definitions for these quantities in non-sinusoidal systems [1]-[7]. Various definitions for power components in non-sinusoidal linear systems [1]–[4], and in non-sinusoidal nonlinear systems [5]–[7] have been proposed so far.

As well as a lack of universal and practical definitions, lack of appropriate measurement technique is also present. Currently available analog meters, designed and calibrated for sinusoidal systems are found to be unsuitable under non-sinusoidal circumstances [8]-[11]. Various algorithms for digital power measurement were proposed [12]–[17], and implemented so far [18]–[20]. However there is no digital algorithm proposed for reactive power measurements in non-sinusoidal systems.

The objective of this paper is to investigate and describe how power components can be measured digitally under sinusoidal and non-sinusoidal circumstances by some of the definitions proposed in [1]-[4].

First, existing algorithms for power measurements are analysed [12]–[14]. It has been found that all of them were originally developed for an analog system model and represent digital approximations to the analog processing schemes. It is shown that they all have the same digital form, which was defined earlier in [21]. This form was used to design algorithms for line parameter measurements with low sensitivity to frequency change as well as new algorithms for power measurements in sinusoidal systems [22]. The form was also applied in non-sinusoidal conditions [23]. This paper takes this digital form as a guide in developing the new approach to the algorithm design.

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Second, the new algorithm design approach is developed. The approach starts with defining an algorithm for digital power measurement as a sequence of computations performed on voltage and current samples in order to obtain power. For the purpose of analyses the process of the computations is considered as two dimensional digital signal processing. The analysis is then performed in transformation domain and a general technique for algorithm design is developed. The technique is used to design the algorithms for measuring power components by various definitions for power in sinusoidal as well as non-sinusoidal linear systems.

The nature of the algorithm design problem is briefly described in the second section. Results of the analysis of the existing algorithms for digital power measurements are stated in the third section. A new design technique is proposed in the fourth section. New algorithms for power measurements are defined next. Finally some test results using field signal recordings are presented.

THE NATURE OF THE ALGORITHM DESIGN PROBLEM

Power components have to be measured according to different suggested definitions. In order to state the definitions and design metering algorithms, an adopted model for current and voltage signals is given first. The suggested definitions of power components, given a form convenient for digital implementation, are presented next.

Signal model

In the steady state conditions, voltage $v(t)$ and current $i(t)$ are periodic over the interval T. Although not purely sinusoidal, they are assumed to contain a finite number of higher order harmonics. This assumption is valid most of the time. Periodicity of voltage and current signals is preserved in non-sinusoidal conditions as well. Due to the digital implementation, the sampling theorem requires band limited frequency spectrum. Finite frequency spectra of the signals are provided by low pass filtering. Therefore, the signals can be represented as a sum of their Fourier components as follows:

$$\begin{aligned} v(t) &= \textstyle\sum_{k=0}^{M} V_k sin(\frac{2\pi k}{T}t + \varphi_k) \\ i(t) &= \textstyle\sum_{k=0}^{M} I_k sin(\frac{2\pi k}{T}t + \psi_k) \end{aligned} \quad (1)$$

Assuming that voltage and current signals are uniformly sampled at frequency $\omega_s = N2\pi T$, where $N = 2M+1$ is chosen according to the Sampling theorem, the above equations have the following digital form:

$$\begin{aligned} v_n &= \textstyle\sum_{k=0}^{M} V_k sin(\frac{2\pi k}{N}n + \varphi_k) \\ i_n &= \textstyle\sum_{k=0}^{M} I_k sin(\frac{2\pi k}{N}n + \psi_k) \end{aligned} \quad (2)$$

Various power definitions are based on different considerations of voltage and current signal relation. Also different applications require measurements of different quantities.

Power Component Definitions

Several definitions proposed for measurements of power components in linear sinusoidal and non-sinusoidal systems are adopted from [1] – [4]. The corresponding expressions presented below are derived from the original definition expressions with voltage and current signals given by (1).

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Active power may be expressed as:

- average power [2], [4]:

$$P_{av} = V_0 I_0 + \sum_{k=1}^{M} \frac{1}{2} V_k I_k cos(\varphi_k - \psi_k) \quad (3)$$

- active power of the fundamental harmonic:

$$P_1 = \frac{1}{2} V_1 I_1 cos(\varphi_1 - \psi_1) \quad (4)$$

Reactive power may be expressed as:

- Budeanu's reactive power [1]:

$$Q_{Bud} = \sum_{k=1}^{M} \frac{V_k I_k}{2} sin(\varphi_k - \psi_k) \quad (5)$$

- Fryze's reactive power [2]:

$$Q_{Fr}^2 = \sum_{k=0}^{M} V_k^2 \sum_{k=0}^{M} I_k^2 - [V_0 I_0 + \sum_{k=1}^{M} \frac{1}{2} V_k I_k cos(\varphi_k - \psi_k)]^2 \quad (6)$$

- Kusters's inductive reactive power [4]:

$$Q_{iK}^2 = \frac{\sum_{k=0}^{M} V_k^2}{\sum_{k=1}^{M} \frac{V_k^2}{k}} [\sum_{k=1}^{M} \frac{1}{2} \frac{V_k I_k}{k} sin(\varphi_k - \psi_k)]^2 \quad (7)$$

- Kusters's capacitive reactive power [4]:

$$Q_{cK}^2 = \frac{\sum_{k=0}^{M} V_k^2}{\sum_{k=1}^{M} k V_k^2} [\sum_{k=1}^{M} \frac{1}{2} k V_k I_k sin(\varphi_k - \psi_k)]^2 \quad (8)$$

- reactive power of the fundamental harmonic:

$$Q_1 = \frac{1}{2} V_1 I_1 sin(\varphi_1 - \psi_1) \quad (9)$$

Besides the problem of defining power components in nonsinusoidal system conditions, there is also a problem of their metering. As mentioned, existing meters designed to operate under sinusoidal conditions lose their accuracy in presence of higher harmonics.

This paper proposes digital measurements of power components. In order to find techniques for digital algorithm design, existing algorithms for digital power measurement have been studied.

ANALYSIS OF THE EXISTING ALGORITHMS

Various algorithms for digital power measurements were proposed [12]–[17], and implemented [18]–[20]. All of them have originally been developed for an analog system model, and represent digital approximations of the analog processing schemes. Having defined digital algorithm for power measurements as a set of computations performed on voltage and current samples, in order to obtain power, direct applications of digital signal processing seems more appropriate.

The existing algorithms have been analysed in order to disclose and formulate the actual digital signal processing they perform. It has been concluded that although derived from different analog prototypes and for different purposes, all the algorithms have the same digital form, which has been taken as a guide for the new approach to the algorithm design. Results of the analysis are given below.

There are merely three different digital algorithms for active and two for reactive power measurement published so far [12]–[14].

Malik and Hope assume that current and voltage signals are purely sinusoidal, and propose the following procedure for active and reactive power measurement [12]. First, voltage and current signals are resolved along the orthogonal reference axes as follows:

$$\begin{aligned} V_d(t) &= \tfrac{1}{T}\textstyle\int_{t-T}^{t} v(\tau) sin\tfrac{2\pi}{T}\tau d\tau = \tfrac{1}{2} V_1 cos\varphi_1 \\ V_q(t) &= \tfrac{1}{T}\textstyle\int_{t-T}^{t} v(\tau) cos\tfrac{2\pi}{T}\tau d\tau = \tfrac{1}{2} V_1 sin\varphi_1 \\ I_d(t) &= \tfrac{1}{T}\textstyle\int_{t-T}^{t} i(\tau) sin\tfrac{2\pi}{T}\tau d\tau = \tfrac{1}{2} I_1 cos\varphi_1 \\ I_q(t) &= \tfrac{1}{T}\textstyle\int_{t-T}^{t} i(\tau) cos\tfrac{2\pi}{T}\tau d\tau = \tfrac{1}{2} I_1 sin\varphi_1 \end{aligned} \quad (10)$$

Then, active power is obtained as the sum of the products of the in-phase components. Reactive power is obtained as the difference of the products of the quadrature components. This is given by the following equations:

$$\begin{aligned} P &= 2(V_d I_d + V_q I_q) = \tfrac{1}{2} V_1 I_1 cos(\varphi_1 - \psi_1) \\ Q &= 2(V_d I_q - V_q I_d) = \tfrac{1}{2} V_1 I_1 sin(\varphi_1 - \psi_1) \end{aligned} \quad (11)$$

It should be noted that quantities $V_d(t)$, $V_q(t)$, $I_d(t)$, $I_q(t)$ are not signals, but numbers independent of time when the integration (10) is performed over the period T. However, time t in the upper and lower integration limits is included to point out that the integration process can start at any time instant.

Digital algorithms retain the same procedure but values (10) are computed digitally as follows:

$$\begin{aligned} V_d(n) &= \tfrac{1}{N}\textstyle\sum_{k=0}^{N-1} v_{n-k} sin\tfrac{2\pi}{N}(n-k) \\ V_q(n) &= \tfrac{1}{N}\textstyle\sum_{k=0}^{N-1} v_{n-k} cos\tfrac{2\pi}{N}(n-k) \\ I_d(n) &= \tfrac{1}{N}\textstyle\sum_{k=0}^{N-1} i_{n-k} sin\tfrac{2\pi}{N}(n-k) \\ I_q(n) &= \tfrac{1}{N}\textstyle\sum_{k=0}^{N-1} i_{n-k} cos\tfrac{2\pi}{N}(n-k) \end{aligned} \quad (12)$$

These equations can be used to express active and reactive power, as given by (11), in terms of voltage and current signal samples:

$$\begin{aligned} P &= \tfrac{2}{N^2} \textstyle\sum_{k=0}^{N-1} \sum_{m=0}^{N-1} v_{n-k} i_{n-m} cos\tfrac{2\pi}{N}(k-m) \\ Q &= \tfrac{2}{N^2} \textstyle\sum_{k=0}^{N-1} \sum_{m=0}^{N-1} v_{n-k} i_{n-m} sin\tfrac{2\pi}{N}(k-m) \end{aligned} \quad (13)$$

Schweitzer assumes that current and voltage signals are purely sinusoidal, and proposes a similar procedure for active and reactive power measurement [13]. The difference is that voltage and current signals are not correlated with Fourier functions $cos\frac{2\pi}{T}t$ and $sin\frac{2\pi}{T}t$ as in (10), but with Walsh functions $sal(\frac{t}{T})$ and $cal(\frac{t}{T})$, as follows:

$$\begin{aligned} S_V(t) &= \textstyle\int_{t-T}^{t} v(\tau) sal\tfrac{\tau}{T}\, d\tau = \tfrac{2T}{\pi} V_1 cos\varphi_1 \\ S_I(t) &= \textstyle\int_{t-T}^{t} i(\tau) sal\tfrac{\tau}{T}\, d\tau = \tfrac{2T}{\pi} I_1 cos\varphi_1 \\ C_V(t) &= \textstyle\int_{t-T}^{t} v(\tau) cal\tfrac{\tau}{T}\, d\tau = \tfrac{2T}{\pi} V_1 sin\varphi_1 \\ C_I(t) &= \textstyle\int_{t-T}^{t} i(\tau) cal\tfrac{\tau}{T}\, d\tau = \tfrac{2T}{\pi} I_1 sin\varphi_1 \end{aligned} \quad (14)$$

Then, active and reactive power are obtained by using the following equations:

$$\begin{aligned} P &= \tfrac{1}{2}(\tfrac{\pi}{2T})^2 (S_V S_I + C_V C_I) = \tfrac{1}{2} V_1 I_1 cos(\varphi_1 - \psi_1) \\ Q &= \tfrac{1}{2}(\tfrac{\pi}{2T})^2 (C_V S_I - S_V C_I) = \tfrac{1}{2} V_1 I_1 sin(\varphi_1 - \psi_1) \end{aligned} \quad (15)$$

It should be noted that quantities $S_V(t)$, $C_V(t)$, $S_I(t)$, $C_I(t)$ are not signals, but numbers independent of time when the integration (14) is performed over the period T. However, time t in the upper and lower integration limits again is included to point out that the integration process can start at any time instant.

Digital algorithms retain the same procedure but values (14) are computed digitally, as follows:

$$\begin{array}{l} S_V(n) = \frac{T}{N}\sum_{k=0}^{N-1} v_{n-k} sal\frac{k}{N} \\ C_V(n) = \frac{T}{N}\sum_{k=0}^{N-1} v_{n-k} cal\frac{k}{N} \\ \\ S_I(n) = \frac{T}{N}\sum_{k=0}^{N-1} i_{n-k} sal\frac{k}{N} \\ C_I(n) = \frac{T}{N}\sum_{k=0}^{N-1} i_{n-k} cal\frac{k}{N} \end{array} \quad (16)$$

These equations can be used to express active and reactive power, as given by (15), in terms of voltage and current signal samples:

$$P = \frac{(sin\frac{\pi}{N})^2}{8}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} v_{n-k} i_{n-m}[sal\frac{k}{N}sal\frac{m}{N} + cal\frac{k}{N}cal\frac{m}{N}]$$

$$Q = \frac{(sin\frac{\pi}{N})^2}{8}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} v_{n-k} i_{n-m}[cal\frac{k}{N}sal\frac{m}{N} - sal\frac{k}{N}cal\frac{m}{N}] \quad (17)$$

Turgel makes a digital algorithm for active power measurement by approximating the fundamental equation for calculating the average power (3) as follows [14]:

$$\begin{array}{rcl} P & = & \frac{1}{N}\sum_{k=0}^{N-1} v_{n-k} i_{n-k} \\ & = & \frac{1}{N}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} v_{n-k} i_{n-m}\delta(k-m) \end{array} \quad (18)$$

where $\delta(k-m)$ is defined as:

$$\delta(k-m) = \begin{cases} 1 & \text{if } k = m \\ 0 & \text{otherwise} \end{cases}$$

This concludes the review of current algorithms for digital power measurements. Equations (13), (17), and (18) have been derived to verify the claim that all of the algorithms have the following common form:

$$\mathcal{P}(n) = \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} v_{n-k} i_{n-m} \quad (19)$$

It should be noted that power, as expressed by the above equation, is a function of time. However, a power component is a constant for a given voltage and current signal pair. Discrete time n in the above equation is to point out that calculation of a power component can be done at any time instant based on N previous voltage and current signal samples.

The conclusion that all of the algorithms have the same digital form suggested the idea to take that form as a starting point in algorithm design, and then by using two dimensional digital signal processing techniques, determine coefficients h_{km} according to a particular power definition.

NEW APPROACH TO ALGORITHM DESIGN

Existing algorithms for digital power measurements have been originally derived for different analog system models. However, they all have the same mathematical form. The form has been recognized as a *bilinear form of voltage and current samples* in [21]. It was used to design algorithms for line parameter measurements with low sensitivity to frequency change [22], as well as new algorithms for power measurements [22], [23].

When measured digitally, power components are actually computed based on voltage and current signal samples. The process of the computation can be looked at as two dimensional digital signal processing, and can be analysed in that manner. Here, active, reactive and apparent power are obtained by means of 2D digital, FIR filters. The support to this approach is the fact that in all current algorithms for digital power measurements this kind of filtering can be recognized, as given by equation (19). As stated above, powers are obtained as outputs of 2D digital FIR filters, connected in special schemes. All the filters are derived from the same filter-prototype, described below.

- Inputs to the prototype, denoted by $u_{n_1 n_2}$, are 2D separable square periodic digital sequences:

$$u_{n_1 n_2} = x_{n_1} y_{n_2} \quad (20)$$

where:

$$\begin{array}{rcl} x_n & = & \sum_{k=0}^{M} X_k sin(\frac{2\pi k}{N}n + \phi_k) \\ y_n & = & \sum_{k=0}^{M} Y_k sin(\frac{2\pi k}{N}n + \theta_k) \end{array} \quad (21)$$

- Unit pulse response of a filter, denoted by $h_{n_1 n_2}$, is a 2D finite extent digital sequence:

$$h_{n_1 n_2} = 0$$
$$\text{if } n_1 < 0 \text{ or } n_2 < 0 \text{ or } n_1 \geq N \text{ or } n_2 \geq N$$

- The prototype is assumed to be a linear shift invariant filter, meaning filter output, denoted by $z_{n_1 n_2}$, can be derived as convolution of its unit pulse response with its input as follows:

$$z_{n_1 n_2} = (h**u)_{n_1 n_2} \quad (22)$$

Thus filter output is also a two dimensional sequence. However, outputs of interest here are those where $n_1 = n_2 = n$. They form a 1D sequence z_{nn}.

Considering the above constraints, the input/output relation (22) for the prototype becomes:

$$\begin{array}{rcl} z_{nn} & = & (h**(xy))_{nn} \\ & = & \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} x_{n-k} y_{n-m} \end{array} \quad (23)$$

Here inputs (voltage and current signal samples) are known, and outputs (power components) are defined. The relation between input, output and unit pulse response in the time domain is given by equation (23). Filter unit pulse responses (algorithms) are to be derived to give defined outputs for known inputs. The task is a deconvolution problem, and is not easy to solve in the time domain. Therefore it has been considered in the frequency domain.

Since signals x_n and y_n are periodic, and $h_{n_1 n_2}$ is of finite extent, a two dimensional Discrete Fourier Transform (DFT) is the most appropriate for analysing the prototype in the frequency domain. Considered as circular convolution, the input/output relation (23) in the frequency domain becomes [24]:

$$\begin{array}{rcl} \tilde{Z}_{pq} & = & \tilde{H}_{pq}\tilde{U}_{pq} \\ & = & \tilde{H}_{pq}\tilde{X}_p\tilde{Y}_q \end{array} \quad (24)$$

where:

$$\tilde{Z} \overset{\text{DFT}}{\longleftrightarrow} z \quad \tilde{H} \overset{\text{DFT}}{\longleftrightarrow} h \quad \tilde{X} \overset{\text{DFT}}{\longleftrightarrow} x \quad \tilde{Y} \overset{\text{DFT}}{\longleftrightarrow} y$$

The filter output z_{nn}, expressed as the inverse DFT, is given by the following expression:

$$\begin{array}{rcl} z_{nn} & = & \frac{1}{N^2}\sum_{p=0}^{N-1}\sum_{q=0}^{N-1}\tilde{Z}_{pq}W_N^{-np}W_N^{-nq} \\ & = & \frac{1}{N^2}\sum_{p=0}^{N-1}\sum_{q=0}^{N-1}\tilde{H}_{pq}\tilde{X}_p\tilde{Y}_q W_N^{n(p+q)} \end{array} \quad (25)$$

In order to be used for measuring power components, which are by all of the suggested definitions constants for given voltage and current signal pair, z_{nn} should also be independent on n. Considering equation (25), this condition imposes the following constraint on the system function $\tilde{H}_{pq}$:

$$\tilde{H}_{pq} = 0 \quad \text{if } p + q \neq rN \quad r \in \{0,1\}$$

Therefore, equation (25) becomes:

$$z = z_{nn} = \frac{1}{N^2}\sum_{p=0}^{N-1}\tilde{H}_{p\,N-p}\tilde{X}_p\tilde{Y}_{N-p} \quad (26)$$

Since sequences x and y are given by (21), their DFT-s $\tilde{X}$ and $\tilde{Y}$ have following values (APPENDIX A):

$$\tilde{X}_p = \begin{cases} NX_0 & p=0 \\ \frac{N}{2j}X_pe^{j\phi_p} & 1 \le p \le M \\ -\frac{N}{2j}X_{N-p}e^{-j\phi_p} & M+1 \le p \le N-1 \end{cases} \quad (27)$$

$$\tilde{Y}_p = \begin{cases} NY_0 & p=0 \\ \frac{N}{2j}Y_pe^{-j\theta_p} & 1 \le p \le M \\ -\frac{N}{2j}Y_{N-p}e^{j\theta_p} & M+1 \le p \le N-1 \end{cases} \quad (28)$$

Substituting values given by (27) and (28), equation (26) becomes:

$$z = \tilde{H}_{00}X_0Y_0 + \frac{1}{4}\sum_{p=1}^{M}(\tilde{H}_{p\,N-p}X_pY_pe^{j(\phi_p-\theta_p)} + \tilde{H}_{N-p\,p}X_pY_pe^{-j(\phi_p-\theta_p)}) \quad (29)$$

The equation (29) in the frequency domain is equivalent to the input/output relation for the filter prototype (23) in the time domain. It is, however, more convenient for the filter designs.

To design a filter means to derive its impulse response so that it gives defined output (a power component) for known input (voltage and current signal samples). A power component is defined in terms of magnitudes and phase angles of voltage and current harmonics. The prototype output is also, by equation (29), defined in terms of magnitudes and phase angles of its input's harmonics. Only outputs that are of concern here are those of the form:

$$z = \alpha_0X_0Y_0 + \sum_{p=1}^{M}[\alpha_p\frac{X_pY_p}{2}cos(\phi_p-\theta_p) + \beta_p\frac{X_pY_p}{2}sin(\phi_p-\theta_p)] \quad (30)$$

where $\alpha_0,\ldots,\alpha_M,\ \beta_1,\ldots,\beta_M$ are arbitrary constants. It is shown in the following section that knowing how to measure quantities of this form is enough to construct algorithms for measuring powers by all of the definitions given above.

It remains to specify the prototype impulse response so that it gives output (30) for input (21). Considering equation (30), the filter system function $\tilde{H}$ should be chosen as follows:

$$\tilde{H}_{00} = \alpha_0$$

$$\tilde{H}_{p\,N-p} = \alpha_p + j\beta_p \quad \tilde{H}_{p\,N-p} = \alpha_p - j\beta_p \quad p=1,\ldots,M \quad (31)$$

Now the filter unit pulse response can be found as inverse 2D DFT of the filter system function as follows:

$$h_{n_1n_2} = \frac{\alpha_0}{N^2} + \frac{2}{N^2}\sum_{p=1}^{M}[\alpha_pcos\frac{2\pi p}{N}(n_1-n_2) + \beta_psin\frac{2\pi p}{N}(n_1-n_2)] \quad (32)$$

Briefly, it has been proved that two dimensional filtering (23) with unit pulse response (32), applied on signals (21), provides measuring quantities of the form (30). It is, however, possible to construct algorithms for measuring the powers by all of the definitions given above by knowing how to measure quantities of the form (30) only.

NEW ALGORITHMS FOR POWER MEASUREMENTS

Some of the power definitions can be derived directly from equation (30) by assigning $x_n = v_n$, $y_n = i_n$ and giving certain values to constants $\alpha_0,\ldots\alpha_M,\ \beta_1,\ldots\beta_M$. These power components can be measured merely by applying filtering (23) on current and voltage samples. The other power definitions such as Fryze's reactive power, defined by (6), cannot be derived from (30) the same way. However, these power components can be measured in a similar manner, by connecting several filter units each performing filtering (23), in certain schemes.

Avarage active power definition (3) can be derived from equation (30) by assigning $x_n = v_n$, $y_n = i_n$, and $\alpha_0 = \ldots = \alpha_M = 1$, $\beta_1 = \ldots = \beta_M = 0$. This component can be obtained by the measuring scheme shown in Figure 1, with (APPENDIX B):

$$h_{n_1n_2} = \frac{1}{N}\delta(n_1-n_2) \quad (33)$$

First harmonic's active power definition (4) can be derived from equation (30) by assigning $x_n = v_n$, $y_n = i_n$, and $\alpha_0 = 0$, $\alpha_1 = 1$, $\alpha_2 = \ldots = \alpha_M = 0$, $\beta_1 = \ldots = \beta_M = 0$. This component can be obtained by the measuring scheme shown in Figure 1, with:

$$h_{n_1n_2} = \frac{2}{N^2}cos\frac{2\pi}{N}(n_1-n_2) \quad (34)$$

First harmonic's reactive power definition (9) can be derived from equation (30) by assigning $x_n = v_n$, $y_n = i_n$, and $\alpha_0 = \ldots = \alpha_M = 0$, $\beta_1 = 1, \beta_2 = \ldots = \beta_M = 0$. This component can be obtained by the measuring scheme shown in Figure 1, with:

$$h_{n_1n_2} = \frac{2}{N^2}sin\frac{2\pi}{N}(n_1-n_2) \quad (35)$$

Budeanu's reactive power definition (9) can be derived from equation (30) by assigning $x_n = v_n$, $y_n = i_n$, and $\alpha_0 = \ldots = \alpha_M = 0$, $\beta_1 = \ldots = \beta_M = 1$. This component can be obtained by the measuring scheme shown on Figure 1, with:

$$h_{n_1n_2} = \frac{2}{N^2}\sum_{k=1}^{M}sin\frac{2\pi}{N}k(n_1-n_2) \quad (36)$$

Fryze's reactive power definition (6) cannot be derived directly from equation (30), but all of the three terms ($\sum_{k=0}^{M}V_k^2$, $\sum_{k=0}^{M}I_k^2$, $V_0I_0 + \sum_{k=1}^{M}\frac{1}{2}V_kI_kcos(\varphi_k-\psi_k)$) that appear in the definition can be derived directly. Term $\sum_{k=0}^{M}V_k^2$ can be derived from equation (30) by assigning $x_n = v_n$, $y_n = v_n$, and $\alpha_0 = \ldots = \alpha_M = 1$, $\beta_1 = \ldots = \beta_M = 0$. Term $\sum_{k=0}^{M}I_k^2$ can be derived from equation (30) by assigning $x_n = i_n$, $y_n = i_n$, and $\alpha_0 = \ldots = \alpha_M = 1$, $\beta_1 = \ldots = \beta_M = 0$. Term $V_0I_0 + \sum_{k=1}^{M}\frac{1}{2}V_kI_kcos(\varphi_k-\psi_k)$ can be derived from equation (30) by assigning $x_n = v_n$, $y_n = i_n$, and $\alpha_0 = \ldots = \alpha_M = 1$, $\beta_1 = \ldots = \beta_M = 0$.

This power component can be obtained by the measuring scheme shown in Figure 2, with:

$$\begin{aligned} h^1_{n_1n_2} &= \tfrac{1}{N}\delta(n_1-n_2) \\ h^2_{n_1n_2} &= \tfrac{1}{N}\delta(n_1-n_2) \\ h^3_{n_1n_2} &= \tfrac{1}{N}\delta(n_1-n_2) \end{aligned} \quad (37)$$

Kusters's inductive reactive power definition (6) cannot be derived directly from equation (30), but all of the three terms ($\sum_{k=0}^{M}V_k^2$, $\sum_{k=1}^{M}\frac{1}{k}V_k^2$, $\sum_{k=1}^{M}\frac{1}{2}\frac{V_kI_k}{k}sin(\varphi_k-\psi_k)$) that appear in the definition can be derived directly. Term $\sum_{k=0}^{M}V_k^2$ can be derived from equation (30) by assigning $x_n = v_n$, $y_n = v_n$, and $\alpha_0 = \ldots = \alpha_M = 1$, $\beta_1 = \ldots = \beta_M = 0$. Term $\sum_{k=1}^{M}\frac{1}{k}V_k^2$ can be derived from equation (30) by assigning $x_n = v_n$, $y_n = v_n$, and $\alpha_k = \frac{1}{k}$ for $k = 1,\ldots,M$, $\alpha_0 = \beta_1 = \ldots = \beta_M = 0$. Term $\sum_{k=1}^{M}\frac{1}{2}\frac{1}{k}V_kI_ksin(\varphi_k-\psi_k)$ can be derived from equation (30) by assigning $x_n = v_n$, $y_n = i_n$, and $\alpha_0 = \ldots = \alpha_M = 0$, $\beta_k = \frac{1}{k}$ for $k = 1,\ldots,M$.

This power component can be obtained by the measuring scheme shown in Figure 3, with:

$$\begin{aligned} h^1_{n_1n_2} &= \tfrac{1}{N}\delta(n_1-n_2) \\ h^2_{n_1n_2} &= \tfrac{2}{N^2}\textstyle\sum_{k=1}^{M}\tfrac{1}{k}cos\tfrac{2\pi}{N}k(n_1-n_2) \\ h^3_{n_1n_2} &= \tfrac{2}{N^2}\textstyle\sum_{k=1}^{M}\tfrac{1}{k}sin\tfrac{2\pi}{N}k(n_1-n_2) \end{aligned} \quad (38)$$

Kusters's capacitive reactive power definition (6) cannot be derived directly from equation (30), but all of the three terms ($\sum_{k=0}^{M} V_k^2$, $\sum_{k=0}^{M} kV_k^2$, $\sum_{k=1}^{M} \frac{1}{2}kV_kI_ksin(\varphi_k - \psi_k)$) that appear in the definition can be derived directly. Term $\sum_{k=0}^{M} V_k^2$ can be derived from equation (30) by assigning $x_n = v_n$, $y_n = v_n$, and $\alpha_0 = \ldots = \alpha_M = 1$, $\beta_1 = \ldots = \beta_M = 0$. Term $\sum_{k=0}^{M} kV_k^2$ can be derived from equation (30) by assigning $x_n = v_n$, $y_n = v_n$, and $\alpha_k = k$ for $k = 1,\ldots,M$, $\alpha_0 = \beta_1 = \ldots = \beta_M = 0$. Term $\sum_{k=1}^{M} \frac{1}{2}kV_kI_ksin(\varphi_k - \psi_k)$ can be derived from equation (30) by assigning $x_n = v_n$, $y_n = i_n$, and $\alpha_0 = \ldots = \alpha_M = 0$, $\beta_k = k$ for $k = 1,\ldots,M$.

This power component can be obtained by the measuring scheme shown in Figure 3, with:

$$\begin{aligned} h^1_{n_1n_2} &= \tfrac{1}{N}\delta(n_1 - n_2) \\ h^2_{n_1n_2} &= \tfrac{2}{N^2}\textstyle\sum_{k=1}^{M} kcos\tfrac{2\pi}{N}k(n_1 - n_2) \\ h^3_{n_1n_2} &= \tfrac{2}{N^2}\textstyle\sum_{k=1}^{M} ksin\tfrac{2\pi}{N}k(n_1 - n_2) \end{aligned} \quad (39)$$

The above derived algorithms provide measurements of power components by all the definitions presented in the second section of this paper. The adopted mathematical formulation (2D digital FIR filters) has been proved quite convenient for deriving new algorithms as well as for analysing the existing ones. However, once a new algorithm is derived, it is possible to implement the required calculations using either 1D or 2D digital filters. Further study of implementation possibilities indicates that an efficient recursive calculation procedure can be utilized (APPENDIX C).

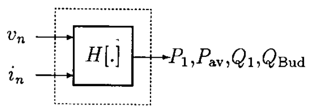

Fig. 1. Measuring Scheme for the Directly Derived Powers

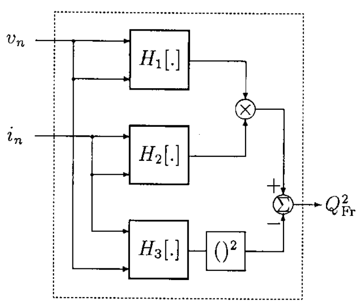

Fig. 2. Measuring Scheme for Fryze's Reactive Power

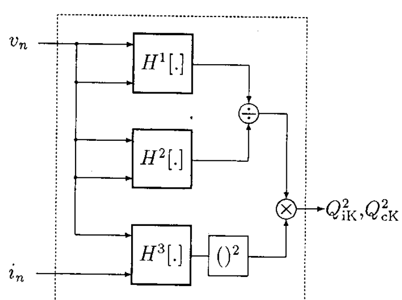

Fig. 3. Measuring Scheme for Kusters's Reactive Powers

EXPERIMENTAL EVALUATION

A FORTRAN computer program has been developed in order to test the proposed algorithms by simulation. The simulation is to verify the design theory, and to perform some of the tests that are required for implementation of the power meters based on digital algorithms. These tests examine the effect of the system frequency change on the accuracy of the algorithms. The algorithms have been tested on two field recorded non-sinusoidal voltage and current pairs.

Test Conditions

The algorithms have been tested on two field recorded non-sinusoidal voltage and current pairs given in [25]. All of the test signals are described in the frequency domain for the reasons given below.

Power components, as presented in the Suggested Definitions section, are defined in terms of magnitudes and phase angles of voltage and current harmonics. Thus a power component for a given voltage and current pair can be computed by its definition if the magnitudes and phase angles of these signals are known. For instance, active power of the first harmonic can be computed by its definition as:

$$P_1 = \frac{1}{2}V_1I_1cos(\varphi_1 - \psi_1) \quad (40)$$

Voltage and current signal samples can be computed based on the magnitudes and phase angles of their harmonics by using equation (2).

An algorithm for digital power measurement is merely a set of computations performed on voltage and current signal samples in order to obtain the power. Thus a power component for a given voltage and current pair can be computed by the corresponding algorithm if the samples of these signals are known. For instance, active power of the first harmonic can be computed by an algorithm as:

$$P_1 = \frac{2}{N^2}\sum_{k=0}^{M}\sum_{m=0}^{M} v_{n-k}i_{n-m}cos\frac{2\pi}{N}(k-m) \quad (41)$$

The power components obtained by the definitions are taken as reference values in tests, and compared with those obtained by applying corresponding algorithms on voltage and current signal samples.

Frequency domain description of the input signals also facilitates simulation of the system frequency change. This is further discussed below.

Tests Description

To verify the design theory means to computationally check formulae and schemes theoretically derived in the previous chapter. The power component obtained by the definitions are taken as reference values and compared with those obtained by applying corresponding algorithms on voltage and current signal samples.

The technique for algorithm design, presented in the previous section, is developed based on the assumption that voltage and current signal samples are of the form:

$$\begin{aligned} v_n &= \textstyle\sum_{k=0}^{M} V_ksin(k\tfrac{2\pi}{N}n + \varphi_k) \\ i_n &= \textstyle\sum_{k=0}^{M} I_ksin(k\tfrac{2\pi}{N}n + \psi_k) \end{aligned} \quad (42)$$

When the system frequency changes, voltage and current signal samples cannot be represented the same way because the ratio between the system frequency and the sampling frequency is not an integer number any more. Strictly speaking, voltage

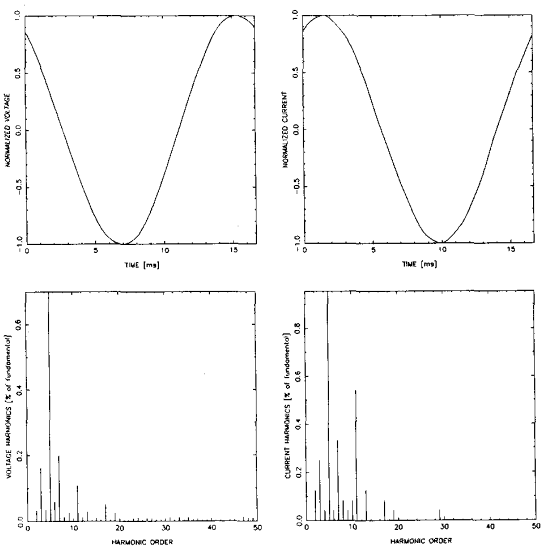

Fig. 4. Voltage and Current Signal Pair W_1

and current signal samples in the new conditions are not N-periodic and DFT does not apply. This is a general problem of digital algorithms for power measurements, and there is a methodology proposed for treating the problem in sinusoidal systems [22].

The tests performed here have been undertaken to indicate the sensitivity of the derived algorithms to small (±0.1% and ±0.5%) but the most common changes of the system frequency. System frequency change is simulated while computing the voltage and current signal samples as follows:

$$\begin{aligned} v_n &= \textstyle\sum_{k=0}^{M} V_k sin(k\frac{2\pi}{N}nc_f + \varphi_k) \\ i_n &= \textstyle\sum_{k=0}^{M} I_k sin(k\frac{2\pi}{N}nc_f + \psi_k) \end{aligned} \qquad (43)$$

where coefficient c_f represent the ratio between the actual system frequency and the nominal system frequency.

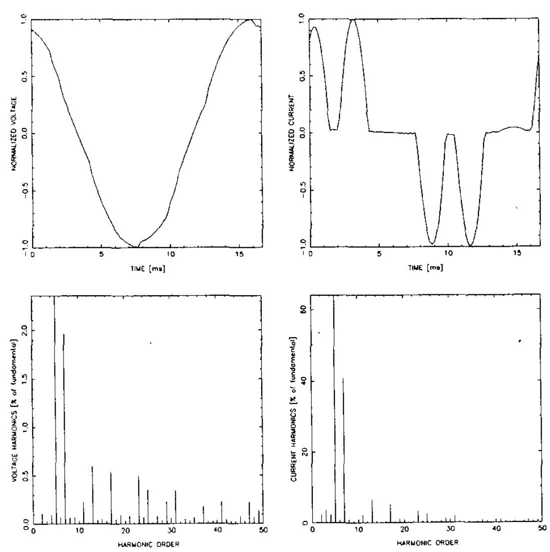

Fig. 5. Voltage and Current Signal Pair W_2

In non-sinusoidal systems, the derived algorithms are to measure power accurately for any voltage and current pair without being affected by their harmonic content. As mentioned earlier, the algorithms have been tested on the set of two recorded signal pairs given in [25]. These signals have various harmonic contents, shown together with corresponding waveforms in the Figures 4 and 5. Since the signals are frequency limited to $M = 50$, the sampling rate is chosen to be $N = 2M + 1 = 101$. Testing the algorithms on two recorded signal pairs with random values of harmonics was to examine their accuracy in conditions of different harmonic contents.

Test Results

A. Base Test

The relative errors for active power computed by the algorithm for measuring the active power of the first harmonic, and by the algorithm for measuring the average power are given in Table I.

Table I. Base Test for Active Power

WAV	P_1	P_{av}
W_1	8.23E-08	7.08E-08
W_2	2.07E-07	1.37E-07

The relative errors for reactive power computed by the algorithm for measuring the reactive power of the first harmonic, Budeanu reactive power, Fryze's reactive power, Kusters's inductive reactive power, and Kusters's capacitive reactive power are given in Table II.

Table II. Base Test for Reactive Power

WAV	Q_1	Q_{Bud}	Q_{Fr}	Q_{iK}	Q_{cK}
W_1	1.62E-08	1.95E-08	8.40E-08	6.36E-08	2.00E-07
W_2	2.99E-08	9.33E-08	1.41E-07	6.48E-08	3.47E-07

Accuracy of the algorithms in ideal conditions is good; the highest relative error was $3.47 \cdot 10^{-7}$ for the algorithm for Kusters's capacitive reactive power measurement. The non-zero errors are due to finite precision of computer number representation. No relation between the errors and harmonic content of the signals have been noticed.

B. Frequency Variation Test

The relative errors for active power computed by the algorithm for measuring the active power of the first harmonic, and by the algorithm for measuring the average power for frequency change of ±0.1% and ±0.5% are given in Table III.

Table III. Frequency Change Test For Active Power

Frequency variation −0.1%		
WAV	P_1	P_{av}
W_1	2.32E-05	2.33E-05
W_2	3.59E-03	3.60E-03
Frequency variation +0.1%		
WAV	P_1	P_{av}
W_1	2.31E-05	2.32E-05
W_2	3.66E-03	3.67E-03
Frequency variation −0.5%		
WAV	P_1	P_{av}
W_1	1.17E-04	1.18E-04
W_2	5.71E-03	4.72E-03
Frequency variation +0.5%		
WAV	P_1	P_{av}
W_1	1.15E-04	1.59E-04
W_2	5.88E-03	1.89E-04

The relative errors for reactive power computed by the algorithm for measuring the reactive power of the first harmonic, Budeanu reactive power, Fryze's reactive power, Kusters's reactive inductive reactive power, and Kusters's capacitive reactive power for frequency change of ±0.1% and ±0.5% are given in Table IV.

Table IV. Frequency Change Test For Rective Power

Frequency variation −0.1%					
WAV	Q_1	Q_{Bud}	Q_{Fr}	Q_{iK}	Q_{cK}
W_1	2.80E-05	2.24E-05	2.00E-05	5.95E-05	7.56E-06
W_2	5.41E-04	4.80E-04	3.75E-04	2.85E-04	2.65E-03
Frequency variation +0.1%					
WAV	Q_1	Q_{Bud}	Q_{Fr}	Q_{iK}	Q_{cK}
W_1	2.10E-05	1.75E-05	1.91E-05	2.16E-05	7.06E-06
W_2	5.48E-04	4.83E-04	4.00E-04	8.03E-04	2.68E-03
Frequency variation −0.5%					
WAV	Q_1	Q_{Bud}	Q_{Fr}	Q_{iK}	Q_{cK}
W_1	2.11E-04	1.60E-04	1.11E-04	1.61E-05	1.13E-03
W_2	2.52E-03	2.33E-03	1.60E-03	2.34E-03	2.64E-03
Frequency variation +0.5%					
WAV	Q_1	Q_{Bud}	Q_{Fr}	Q_{iK}	Q_{cK}
W_1	3.49E-04	4.09E-04	8.57E-05	7.25E-05	8.68-04
W_2	2.77E-03	2.42E-03	2.21E-03	2.79E-03	1.05E-03

Accuracy of the algorithms in conditions of system frequency change is in all of the cases worse than the error of the corresponding algorithms for sinusoidal case. The highest relative error was 0.00277% for test pair W_2, algorithm for average power measurement, and frequency variation of +0.5%. The non-zero errors are due to finite precision of computer number representation and the system frequency change. The highest error occurred for the algorithm for average power measurement because it treats all of the harmonics evenly and the frequency change becomes more significant for higher harmonics. For a particular value of frequency variation the higher error occurred for the second test pair for each of the algorithms. This is because current signal 2 is highly distorted; the fifth harmonic is 65% of the fundamental, and the seventh harmonic is 40% of the fundamental.

Briefly, the above tests have shown that accuracy of the algorithms in ideal conditions is satisfactory, and independent of the harmonic content of the voltage and current signals. In presence of the system frequency change, accuracy of the algorithms decreases with the signal distortion.

CONCLUSIONS

Power components are to be measured under sinusoidal and non-sinusoidal circumstances. Currently available analog meters are unsuitable for non-sinusoidal systems whereas digital power meters are designed to measure either active and reactive power in sinusoidal systems or active power only in non-sinusoidal systems.

This paper has derived and tested algorithms for digital measurements of power components according to the suggested definitions in sinusoidal and non-sinusoidal linear systems. The most important contribution of this study is the proposed algorithm design technique. The technique is general, based on two dimensional digital signal processing.

Applying this technique in deriving the algorithms for measuring first harmonic active and reactive power, as well as for measuring average power, led to derivation of the known algorithms introduced earlier by Malik and Hope, and Turgel. Several new algorithms for measurement of the quantities for which there are no proposed digital algorithms, have been derived by using the new technique. These applications have demonstrated that derivation of algorithms for digital measurements of power components is straight-forward when the proposed technique is used.

The technique is not limited to the extent of designing the algorithms for present needs only, but it may be used in designing the algorithms for some new applications as well. All of the derived algorithms can be implemented as 2D digital FIR filters connected in special schemes. This implementation is suitable for the use of custom designed chips. There are several computational advantages of this implementation approach which are now being investigated, and will be reported in the future.

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REFERENCES

[1] C. I. Budeanu, "Puissances Reactives et Fictives," Institut Romain de l'Energie, Publ. 2, Bucharest, Romania, 1927.

[2] S. Fryze, "Active, Reactive and Apparent Power in Non-sinusoidal Systems," *Przegled Elektrotek.* No 7, pp. 193–203, 1931.

[3] C. H. Page, "Reactive Power in Non-sinusoidal Situations," *IEEE Transaction on Instrumentation and Measurement*, vol. IM-29, Dec. 1980.

[4] N. L. Kusters and W. J. M. Moore, "On the Definition of Reactive Power Under Non-sinusoidal Conditions," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-99, pp. 1845–1854, Oct. 1980.

[5] W. Shepherd and P. Zakikhani "Suggested Definition of Reactive Power for Non-sinusoidal Systems," *IEE Proceedings*, vol. 119, pp. 1361–1362, Sept. 1972.

[6] D. Sharon, "Reactive Power Definition and Power Factor Improvement in Nonlinear Systems," *IEE Proceedings*, vol. 120, pp. 704–706, June 1973.

[7] L. S. Czarnecki, "Orthogonal Decomposition of the Currents in 3-phase Nonlinear Asymmetrical Circuit with a Non-sinusoidal Voltage Source," *IEEE Transactions on Instrumentation and Measurement*, vol. 37, pp. 34, March 1988.

[8] A. J. Baggot, "The Effect of Waveshape Distortion on the Measurements of Energy Tariff Meters," *IEE Metering, Apparatus and Tariffs for Electricity Supply*, Conference Publication No. 156. pp. 280–284, London, 1977.

[9] A. Spalti, "Apparent Energy Metering in Three Phase Networks," *IEE Metering, Apparatus and Tariffs for Electricity Supply*, Conference publication No. 156. pp. 285–286, London, 1977.

[10] R. Arsenau and P. S. Filipski, " Effects of Harmonics on Watthour and Demand Meters," *CEA meeting*, Calgary, October 1985.

[11] M. D. Cox and T. B. Williams "Induction Varhour and Solid-State Varhour Meters Performances on Nonlinear Loads," *IEEE PES Winter Meeting*, Paper No. 89WM 049-8 PWRD, 1989.

[12] G. S. Hope, O. P. Malik and J. Chang, "Microprocessor Based Active and Reactive Power Measurement," *Journal of Electric Power and Energy Systems*, vol. 3, April 1981.

[13] E. O. Schweitzer, III, "Microprocessor Algorithm for Revenue Metering Offers High Accuracy and Minimal Computational burden," *The Fifth Annual Western Protective Relay Conference*, Oct. 1978.

[14] R. F. Turgel, "Digital Wattmeter Using a Sampling Method," *IEEE Transaction on Instrumentation and Measurement*, vol. IM-23, pp. 337–341, Dec. 1974.

[15] C. H. Dix, "Calculated Performance of a Digital Sampling Wattmeter Using Systematic Sampling," *IEE Proceedings*, vol. 129, pp. 172–175, May 1982.

[16] F. J. J. Clarke and J. R. Stockton, "Principles and Theory of Wattmeters Operating on the Basis of Regularly Spaced Sample Pairs," *Journal of Physics and Scientific Instruments* vol. 15, pp. 645–652, June 1982.

[17] G. N. Stenbakken, "A Wide-band Sampling Wattmeter," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-103, pp. 2919–2926, Oct. 1984.

[18] A. C. Corney and R. T. Pullman, "Digital Sampling Laboratory Wattmeter," *IEEE Transactions on Instrumentation and Measurement*, vol. IM-36 pp. 54–59, March 1987.

[19] V. V. Athani, "Power System Measurements Using Microprocessors," *IETE Technical Review*, vol. 2, No. 9, pp. 302–306, 1985.

[20] J. J. Hill and W. E. Alderson, "Design of a Microprocessor Based Digital Wattmeter," *IEEE Transaction on Industrial Electronics and Control Instrumentation*, vol. IECI-28, No. 3, pp. 180–184, Aug. 1981.

[21] B. Perunicic, M. Kezunovic, S. Kreso "Bilinear Form Approach to Synthesis of a Class of Electric Circuit Digital Signal Processing Algorithms," *IEEE Transactions on Circuits and Systems*, vol 35, No 9, Sept. 1988.

[22] B. Perunicic, M. Kezunovic, S. Levi, E. Soljanin, "Digital Signal Processing Algorithms for Power and Line Parameter Measurements with Low Sensitivity to Frequency Change," *Proceedings of the IEEE PICA Conference*, Seattle, U.S.A May 1989.

[23] B. Perunicic, M. Kezunovic, S. Levi, E. Soljanin, "Digital Metering of Active and Reactive Power in Non-sinusoidal Conditions Using Bilinear Forms of Voltage and Current Samples," *IEEE Symposium on Networks, Systems and Signal Processing*, Zagreb, Yugoslavia, June 1989.

[24] D. E. Dudgeon and R. M. Mersereau "*Multidimensional Digital Signal Processing*," Englewood Cliffs, Prentice Hall, NY, 1984.

[25] R. Arsenau et al., "Optimum Metering Systems for Loads with High Harmonics Distortion," *CEA report* No 043-D-610, Canada, May 1988.

APPENDIX A

The DFT of a $N-$periodic digital sequence x_n is defined by:

$$\tilde{X}_p = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi pn}{N}} \qquad (44)$$

Since x_n is given by (21) the above equation becomes:

$$\begin{aligned} \tilde{X}_p &= \textstyle\sum_{n=0}^{N-1}[\sum_{k=0}^{M} X_k sin(\frac{2\pi k}{N}n + \phi_k)]e^{-j\frac{2\pi pn}{N}} \\ &= \textstyle\sum_{k=0}^{M} X_k[\frac{e^{j\phi_k}}{2j}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-p)n} - \frac{e^{-j\phi_k}}{2j}\sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k+p)n} \\ &= \begin{cases} NX_0 & p = 0 \\ \frac{N}{2j}X_p e^{j\phi_p} & 1 \le p \le M \\ -\frac{N}{2j}X_{N-p}e^{-j\phi_p} & M+1 \le p \le N-1 \end{cases} \end{aligned} \qquad (45)$$

APPENDIX B

For $\alpha_0 = \ldots = \alpha_M = 1,\ \beta_1 = \ldots = \beta_M = 0.$ equation (32) becomes:

$$\begin{aligned} h_{n_1n_2} &= \textstyle\frac{1}{N^2} + \frac{2}{N^2}\sum_{p=1}^{M} cos\frac{2\pi p}{N}(n_1 - n_2) \\ &= \textstyle\frac{1}{N^2} + \frac{1}{N^2}\sum_{p=1}^{N-1} cos\frac{2\pi p}{N}(n_1 - n_2) \\ &= \textstyle\frac{1}{N^2}\sum_{p=0}^{N-1} cos\frac{2\pi p}{N}(n_1 - n_2) \\ &= \frac{1}{N}\begin{cases} 1 & \text{if } n_1 = n_2 \\ 0 & \text{otherwise} \end{cases} \\ &= \frac{1}{N}\delta(n_1 - n_2) \end{aligned} \qquad (46)$$

APPENDIX C

Expression (19) can be rewritten as:

$$\begin{aligned} P_n = & \textstyle\sum_{k=1}^{N-1}\sum_{m=1}^{N-1} h_{km}v_{n-k}i_{n-m} + \\ & \textstyle\sum_{k=0}^{N-1} h_{k0}v_{n-k}i_n + \sum_{m=0}^{N-1} h_{0m}v_n i_{n-m} - h_{00}v_n i_n \end{aligned} \qquad (47)$$

Equation (32) implies $h_{k+Nm} = h_{km+N} = h_{km}$, and $h_{k+l\,m+l} = h_{km}$. Therefore:

$$\begin{aligned} P_{n-1} = & \textstyle\sum_{k=1}^{N-1}\sum_{m=1}^{N-1} h_{km}v_{n-k}i_{n-m} + \\ & \textstyle\sum_{k=0}^{N-1} h_{k0}v_{n-k}i_{n-N} + \sum_{m=0}^{N-1} h_{0m}v_{n-N}i_{n-m} \\ & + h_{00}v_{n-N}i_{n-N} - h_{00}v_{n-N}i_n - h_{00}v_n i_{n-N} \end{aligned} \qquad (48)$$

Then from (47) and (48) follows:

$$\begin{aligned} P_n - P_{n-1} = & \textstyle(i_n - i_{n-N})\sum_{k=0}^{N-1} h_{k0}v_{n-k} + \\ & \textstyle(v_n - v_{n-N})\sum_{m=0}^{N-1} h_{0m}i_{n-m} - \\ & h_{00}(v_n - v_{n-N})(i_n - i_{n-N}) \end{aligned} \qquad (49)$$

Thus the innovation of P_n needs only four 1D linear filtering operations, three multiplications for active power, and two multiplications for reactive power ($h_{00} = 0$). This is comparable with other existing methods.

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Slobodan Levi received Dipl. Ing. and MS. degrees from University of Sarajevo, Yugoslavia, all in electrical engineering in 1977 and 1988, respectively. He worked for several companies in Yugoslavia holding different responsibilities related to engineering design and production line automation. He is currently a graduate student at the University of Sarajevo working towards his PhD. degree in the area of digital signal processing for power system applications.